Technical Note

Modelling the Dynamics of a Three-wheeled Racecar: A Pilot Study to Establish the Feasibility of Developing a 'Delta' Configuration Performance Car

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Abstract: This paper presents a preliminary study of the dynamics of a 'delta' configuration three-wheeled sports car, based on a Reliant Robin chassis. Stiffness and damping values to give this vehicle 'sporty' ride dynamics have been proposed. Other dynamic qualities such as rollover and steady-state cornering have also been investigated, and then verified using Carmaker® simulation software. The car performs far better than the Robin in steady-state analyses, but simulation indicates that transient maneuvers such as braked corners can still cause the vehicle to roll. Remedial actions such as lowering the center of gravity are suggested. However, much of the data used in these analyses is assumed, and should be updated as the design progresses. Since the standard calculations used in this report are intended for four-wheeled vehicles, there is scope to develop a more detailed transient model to fully describe the unique dynamics of three-wheeled vehicles and develop design guidelines more suited to them.

Keywords: vehicle; dynamics; handling; ride; simulation; bond graphs; bond-graphs

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1. Introduction

Electric cars are an active area of research and development, offering personal mobility with higher efficiency and lower emissions than existing internal combustion engine vehicles [1]. MIBRID Ltd. – an Electric Vehicle company situated in Lincolnshire - are developing a three-wheeled electric sports car called the Mayfly, shown in Figure 1. It is designed to be 'low-cost, stylish, strong, and light … suitable for the unique roads of Britain' [2] and it has the following specifications:

- Plug-in electric vehicle
- Hybrid capacitor/li-ion battery system
- Three wheels with three independent motors (3 x 10kw approx. 40bhp)
- Lightweight composite body (under 500kg)
- Two seats
- Hatchback with room for the everyday.

Model-based design offers the opportunity to investigate the dynamics of the car – and the effects on comfort and handling – while the design is still at an early, fluid stage. Using computer modelling techniques allows the design to be optimized before investing in physical prototype vehicles, and minimizes the risk of the final product having unforeseen undesirable qualities. In the case of the Mayfly, there are two immediate areas for concern:

- The use of in-hub motors yields a higher 'unsprung mass' than seen in conventional cars, which can adversely affect handling.
- Three-wheeled vehicles are associated with rollover, tipping, and 'spinning out' in turns.

This report therefore details the development of handling models and proposes adjustments to the suspension, tyres and weight distribution to yield performance appropriate to a sports car.

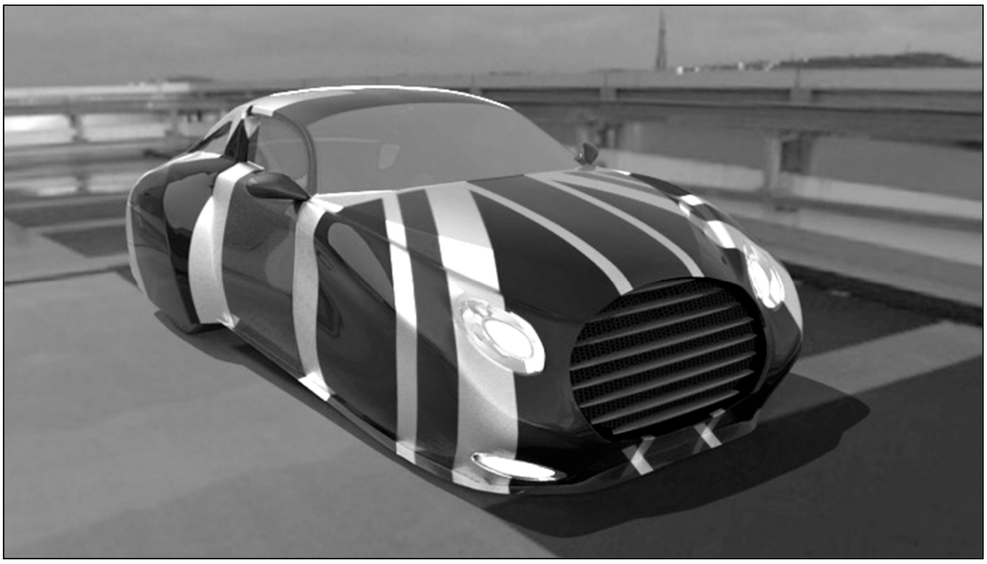

Figure 1. The Mayfly [2].

2. Background

2.1. Three-Wheeled Vehicles

Three-wheeled cars can take one of two configurations: the 'delta' (one wheel at the front and two at the back) or the 'tadpole' (two wheels at the front and one at the back), shown in Figure 2. They can be highly economical: using three wheels instead of four can significantly reduce the weight of a vehicle, improving its power consumption. For 'delta' cars, using one wheel at the front for steering also simplifies the steering system, further reducing weight and cost. 'Tadpole' cars can be extremely aerodynamic.

Three-wheelers are generally associated with instability, in particular rollover (although they can be very stable) [3]. This is particularly true of the 'delta' configuration, which is associated with rolling in braked turns or 'spinning out' when handled roughly, although the tendency to rollover has been exaggerated in popular culture [4]. The 'tadpole' configuration is much more stable [5].

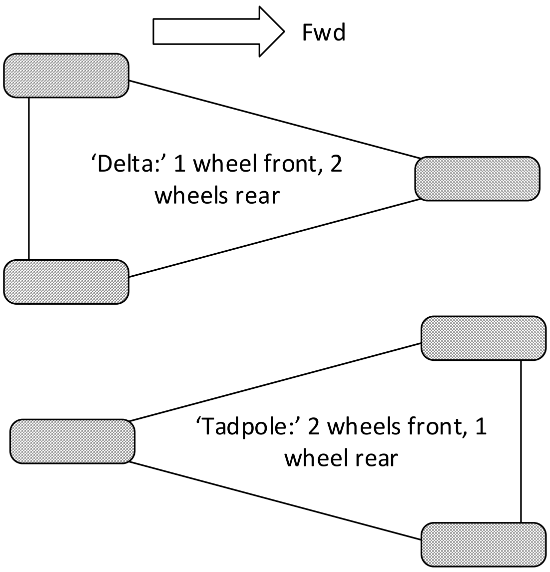

Figure 2. Configurations of Three-wheeled Vehicles.

 doi:10.20944/preprints201805.0074.v1

Dynamically, three-wheeled cars behave like a cross between a motorbike and a car [6]. A tadpole car behaves like a motorbike at the rear (where a single wheel is connected to the drivetrain) and a car at the front (where two wheels usually steer), allowing it to perform as a more stable motorbike. Delta cars, however, behave like a car at the rear (typically connected to the drivetrain) but a motorbike at the front, which is where the steering occurs. A motorcyclist would lean into corners counteracting the lateral forces on the bike by wheel camber action as well as tire slip. This does not occur in a three-wheeled car, where the chassis generally does not allow the front wheel to tilt significantly. Narrow-track three-wheel vehicles have notoriously limited rollover stability, and there is a body of work on 'tilting three-wheelers' where the vehicle leans into corners in the same way that a motorcyclist leans [6]. 'Tilting' three wheeled cars such as the GM Green Machine [7] or CLEVER [3,6,8] therefore improve stability by allowing the wheel(s) to lean as a motorbike would.

2.2. The Reliant Robin

The initial prototype is based on a Reliant Robin chassis. The Robin is a small 3-wheeled car in the 'delta' configuration, which was produced in the UK between 1973 and 2002 and still has a cult following.

The rear suspension utilizes semi-elliptic leaf springs and a DeDion axle [9]. The DeDion 'tube' has a sliding joint to permit wheel track variation during suspension movement, but it keeps both wheels parallel to each other under all conditions (so they are always perpendicular to the road surface regardless of body roll, and hence more stable). This system is cheaper than most independent suspensions, and offers superior handling [10]. The use of the DeDion tube means that the vehicle has much less unsprung weight than a fixed axle would because the final drive / differential and driving shafts are not rigidly attached to the wheels. The system is a popular choice for budget sports cars and coupes.

The front suspension of the Reliant Robin is a leading arm with coil spring damper unit [11]. With this design, the wheel remains parallel to the body and cambers with roll [10].

In producing the models, data for the 1981 Mk 1 Reliant Robin was assumed as a starting point. The Robin evolved somewhat over the years, including plans for an electric Robin prior to production ceasing [12].

Note that the Robin is rear wheel drive and the front wheel steers. The Mayfly has a hub motor on each wheel, making it all-wheel-drive and offering the possibility for differential steering on the rear wheel.

2.3. Electric Vehicles

The prototype is a fully electric BEV (Battery Electric Vehicle) with three hub motors i.e. one in each wheel. This configuration has already been set: electric vehicles can have a variety of configurations such as a single motor and transmission, or hub motors in only some wheels [13].

The weight distribution will therefore be significantly different to that of the Robin, since the conventional engine and drivetrain are replaced by the battery and hub motors as indicated in figure 3. The battery is assumed to be in a low central position, but could again be positioned in place of the engine or in the boot.

3. Method

3.1. Modelling Approach

In constructing a model, a 'crawl-walk-run' approach is followed whereby a simplistic, single degree-of-freedom initial model is created, analyzed, and then complexity is added to this model in stages. In the case of a car, the ride dynamics (symmetric motions) are first investigated using the 'Quarter Car' model. Side forces, slip and responsiveness are then investigated via the 'Bicycle Model' to establish over/under-steer. Quasi-static rigid rollover is also assessed. The results are verified by simulation, and should eventually be validated against physical tests.

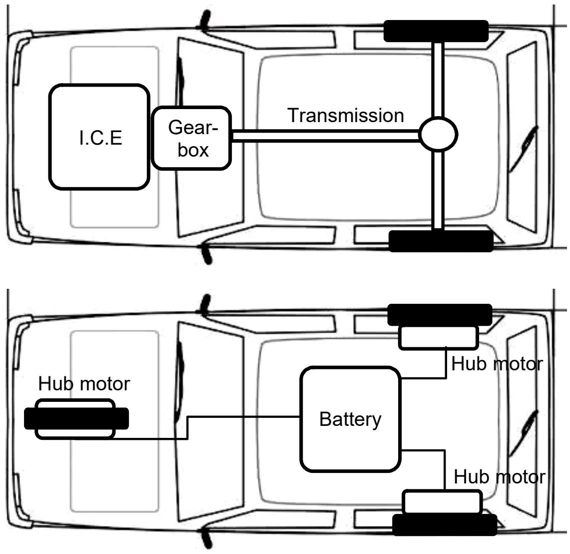

Figure 3. The Robin's drivetrain [top] and the prototype electric drivetrain [lwr].

Additional analysis such as those for 'wobble' (seen on three-wheeled rickshaws) [14] are neglected here.

3.2. Ride Dynamics: The Quarter Car Model

The standard initial model used for modelling vehicle dynamics is the quarter car, shown in figure 4. This models the vertical motion of each suspension unit under the effective load of the body. The suspension is abstracted to a linear spring and damper acting in the vertical sense only, and the tire is also abstracted to a linear vertical spring (and sometimes a damper). The effective load on the system from the vehicle (the 'sprung mass') and the weight of the wheel, suspension, hubs and axle (the 'unsprung mass') are represented as rigid bodies. For the purposes of this project, the model was constructed as a bond graph in 20Sim® software.

'Starting values' for the suspension stiffness and damping were calculated to give desired ride frequency of 2Hz (sprung mass), 15Hz (unsprung mass), which are typical for a 'sports' car [15]. Two inches of travel was assumed and a damping ratio of 0.5. This is in stark contrast to passenger vehicles designed according to the 'Olley Criteria' [10] with softer suspension, to give increased comfort at the expense of handling. The model was then optimized to give a slightly underdamped response, which is ideal for handling.

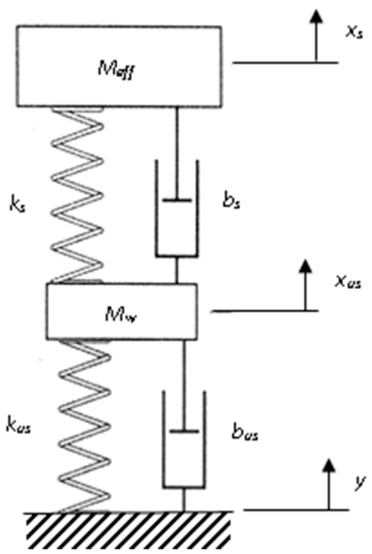

Figure 4. The Quarter Car Model.

Two quarter car models can be connected by a rigid vehicle body to yield a half car model, allowing pitch to be modelled in addition to vertical motion. The front suspension should have a 30% lower ride rate than the rear suspension [10].

3.3. Handling Dynamics: The Bicycle Model

For the purposes of cornering, the car is assumed to have rigid axles and can be simplified to a 'bicycle' i.e. one wheel acting along the center of gravity where there are two in reality [10,16], as shown in figure 5. This type of analysis can be applied to the three-wheeled car: there is already a single wheel at the front (for the delta configuration), and the rear axle is simplified.

The suspension is assumed to be rigid in this model, and gyroscopic effects are ignored. This is because the dominant mechanism for counteracting lateral forces generated in cornering [in a car] is tire slip.

The steer angle required to negotiate a curve with radius R is given by [17]:

$$\delta = \frac{L}{R} + (\alpha_f - \alpha_r), \tag{1}$$

Where slip angles α_f and α_r are functions of mass, lateral acceleration and tire stiffness. At low speeds, α_f and α_r cancel each other out, and δ is the Ackermann angle. At higher speeds, tire slip increases.

Understeer Gradient K (a function of mass and tire stiffness) is given by:

$$K = \frac{W_f}{C_{\alpha f}} - \frac{W_r}{C_{\alpha r}}, \tag{2}$$

Where W_f and W_r are the front and rear weights respectively, and $C_{\alpha f}$ and $C_{\alpha r}$ are the lateral slip stiffness' (sometimes called cornering stiffness').

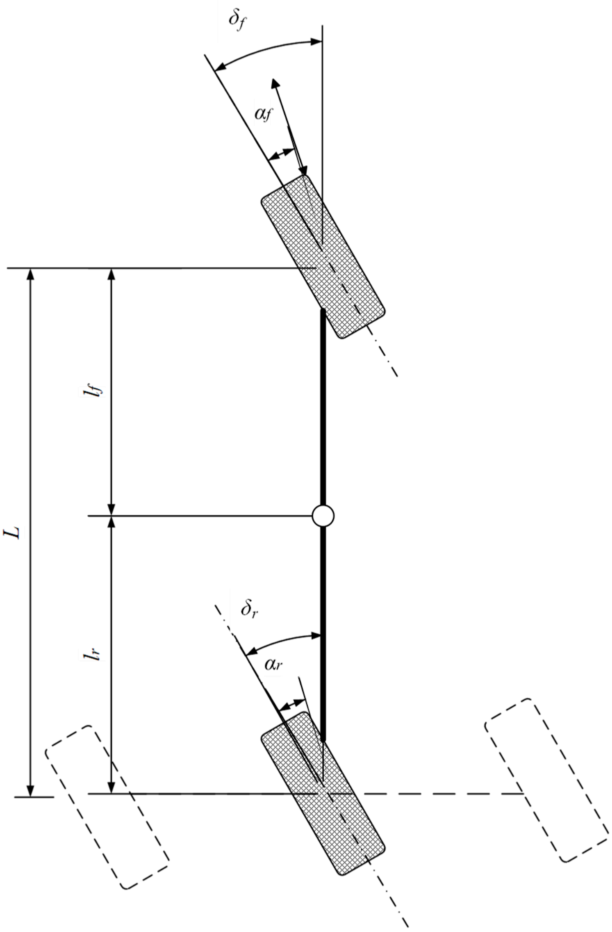

Figure 5. Schematic of the 'Bicycle' model.

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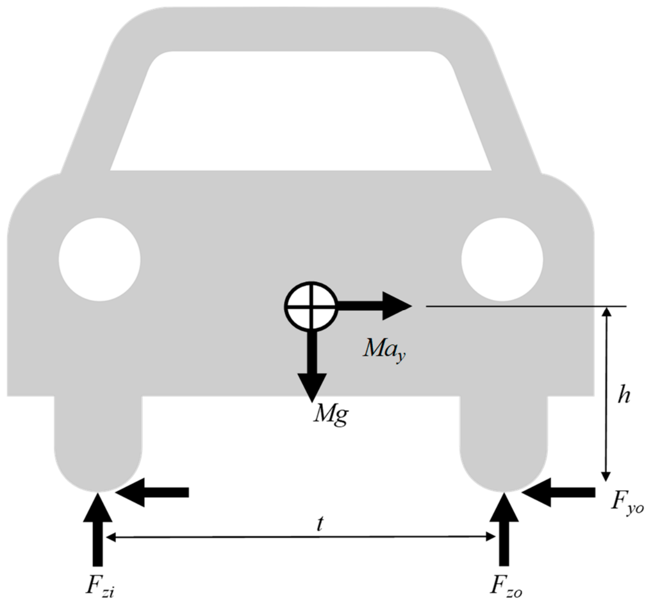

Figure 6. The Quasi-Static Rigid Rollover model.

Note that under braking, the lateral force at a given slip angle is reduced [10] and the steady-state model no longer holds true.

K>0 implies understeer: i.e. vehicle is more stable, but less responsive at high speed, while *K*<0 implies oversteer: i.e. vehicle is more responsive, but has poor high speed stability. For 4WD, the "rear axle should overdrive the front axle to ensure understeer behavior" [18].

3.4. Rollover: The Quasi-StaticRigid Rollover Model

Rollover occurs when lateral accelerations on the vehicle exceed what the tires can compensate for [10]. This is clearly connected to steering and cornering, which is investigated using quasi-static analysis for steady cornering. It can also occur when the vehicle is maneuvering on a cambered surface or experiences a disturbance (e.g. hitting a pavement) – situations analyzed using a transient analysis.

The simplest analysis is the Quasi-Static Rollover of a rigid vehicle i.e. neglecting the deflections of the suspension and tires. On a flat surface, the rollover threshold a_y/g is given by:

$$\frac{a_y}{g} = \frac{t}{2h}, \tag{3}$$

Where t is the 'tread' (lateral distance between wheels) and h is the height of the center of gravity, as indicated in figure 6. This analysis presents a problem for the three-wheeled car, where there is a finite tread at the rear but a zero tread (i.e. one wheel) at the front. Using the rear value therefore gives an optimistic result. Watching delta vehicles such as the Robin roll in practice reveals that they actually roll diagonally in a combined roll/pitch movement [19], which does not appear to have been explicitly tackled in the literature.

For a 4-wheel vehicle, the rigid rollover model overestimates the rollover threshold [10]. This simplistic model can be improved by considering the defections of the suspension and tires, to give the Quasi-Static Suspended Model. This essentially takes account of the center of gravity becoming offset during a maneuver as the vehicle rolls. Due to the complex geometries involved, this model is generally simulated by computer rather than solved analytically, but it is interesting to note that the rollover threshold is generally reduced by 5% compared to the rigid model, and less so for sports cars with a low center of gravity. The DeDion axle at the rear of the Robin chassis should further reduce roll [20]. The most complete model is a Yaw-Roll model, which accounts for the lateral accelerations produced by yawing motions in addition to those produced by cornering and roll [10].

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3.5. Verification by Simulation

The models were verified using IPG Carmaker® [21]: an industry-standard automotive simulation package which allows a virtual prototype to be constructed and tested. This type of full vehicle model allows all possible dynamic behavior and their interactions (which can be significant) to be simulated together.

It should be remembered that this model still includes modelling assumptions and generic data: validation using a physical prototype is recommended.

4. Results & Discussion

4.1. Ride Dynamics: The Quarter Car Model

The optimization yielded the parameters in table 1. With these parameters, the system responds well (slightly underdamped) to a step input as shown in figure 7. These parameters apply to the front suspension, and the rear suspension should be slightly stiffer with a 30% lower ride rate [15].

Table 1. Optimized suspension parameters.

Parameter	Optimized value	Units
K_s	32742.0	N/m
K_{us}	228000.0	N/m
b_s	2084.3	N.s/m
b_{us}	410.0	N.s/m

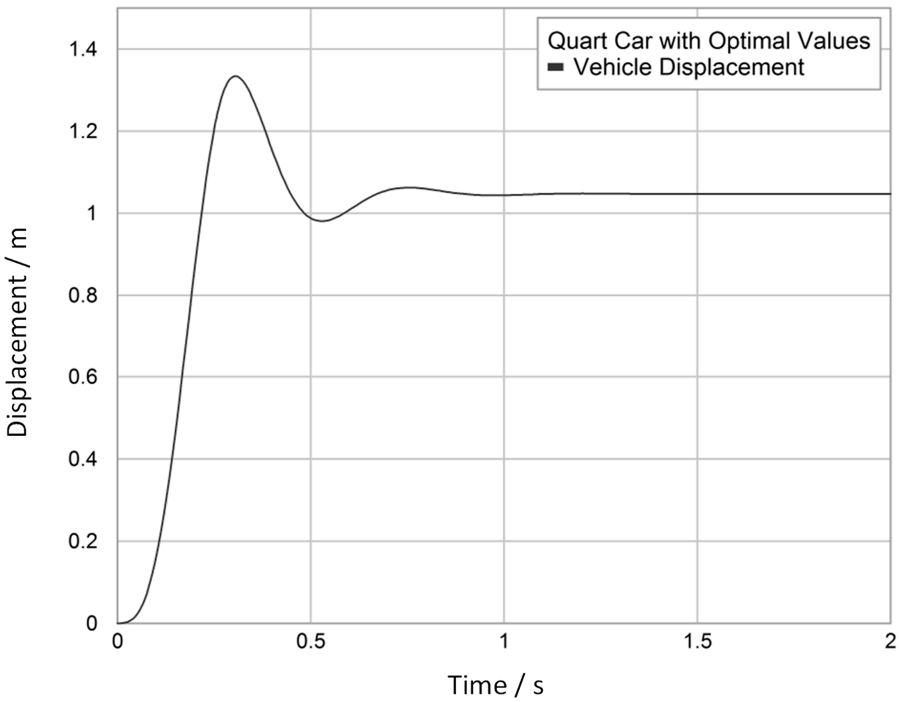

Figure 7. Step response of the Quarter Car populated with optimized values.

4.2. Steady-State Cornering: Bicycle Model

Understeer gradient is given by equation (2). Assuming a slightly rearwards weight distribution and typical lateral stiffness values for the tires, the understeer gradient for the Mayfly can be calculated as 0.037.

The positive understeer gradient *K* indicates that the Mayfly will understeer. All production cars have some degree of understeer built in for safety, but the Mayfly understeers more than the Robin (*K* = 0.026) meaning that it is more stable but less responsive. In addition, the Mayfly is all-wheel-drive, which tends to further increase understeer [10].

Performance can be improved by using larger wheels and stiffer tires, shifting the center of gravity further rearwards, and/or steering the rear wheels. There is also the possibility to use Torque Vectoring or Intelligent Driveline Design (IDD) [22].

Note that delta vehicles with understeer and a more forward center of gravity can be prone to tipping. Adjusting the weight distribution rearwards can help.

4.3. Rollover: The Quasi-StaticRigid Rollover Model

Robin's center of gravity is slightly rearwards (44% weight distribution at front), and the height is roughly equal to that of the engine's camshaft [9] approximately 0.445m. Mayfly's Centre of Gravity is much lower at ~0.241m, because the hub motors and battery are lower than the engine and fuel tank. It is fairly central, but this could be adjusted by repositioning the battery (which is assumed to be in a low central position) [13].

Mayfly's Rollover threshold is 1.2g. This is a significant improvement on Robin (~0.6g). It compares favorably to a typical sports car, which has a center of gravity at *h*=18-20", a tread of *t*= 50-60", and consequently a rollover threshold of 1.2-1.7g. However, it should be noted that this calculation is intended for a 4-wheel car. The delta car has a significantly wider track at the rear (used in this calculation) than at the front (where the single wheel implies a track width of zero), and can be expected to rollover diagonally and more readily than the equivalent four-wheeled car.

Rollover performance can be improved by further lowering the Centre of Mass and widening the track.

4.4. Simulation

Mayfly successfully completes standard tests such as ISO Lane Change and 18m slalom, while Robin tips over. However, Mayfly does tip while cornering at moderate velocities of around 12m/s (28mph) as shown in figure 8. Mayfly is unable to complete a lap of the Nordschliefe circuit (figure 9, 10), which would be a reasonable level of performance for a sports car. This is hardly surprising, as the basic calculations in this report are for steady-state cases whereas the simulation shows the vehicle rolling in transient maneuvers (e.g. braking in a corner), under which case it is well known that the lateral force at a given slip angle is reduced [10]. Tilting the front wheel could increase lateral force through camber (like a motorbike) and remedy this issue.

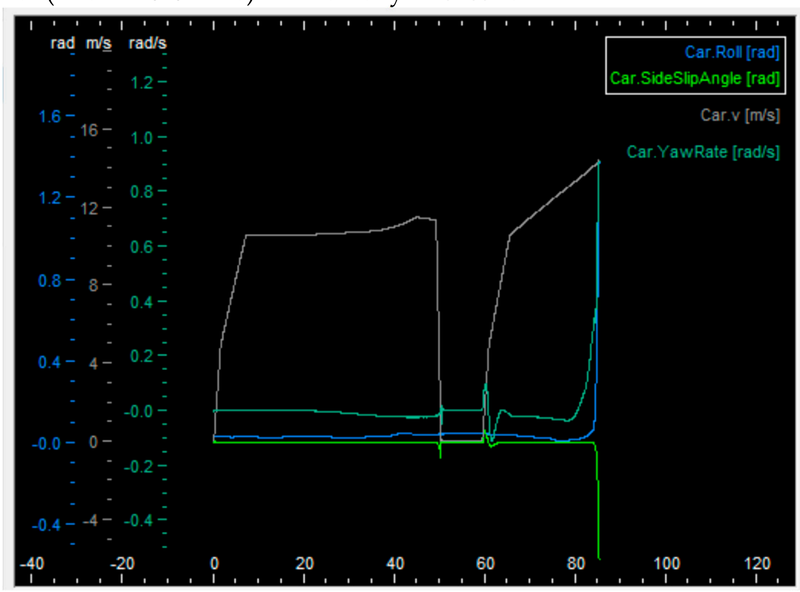

Figure 8. Carmaker® Simulation results showing the Mayfly during transient cornering.

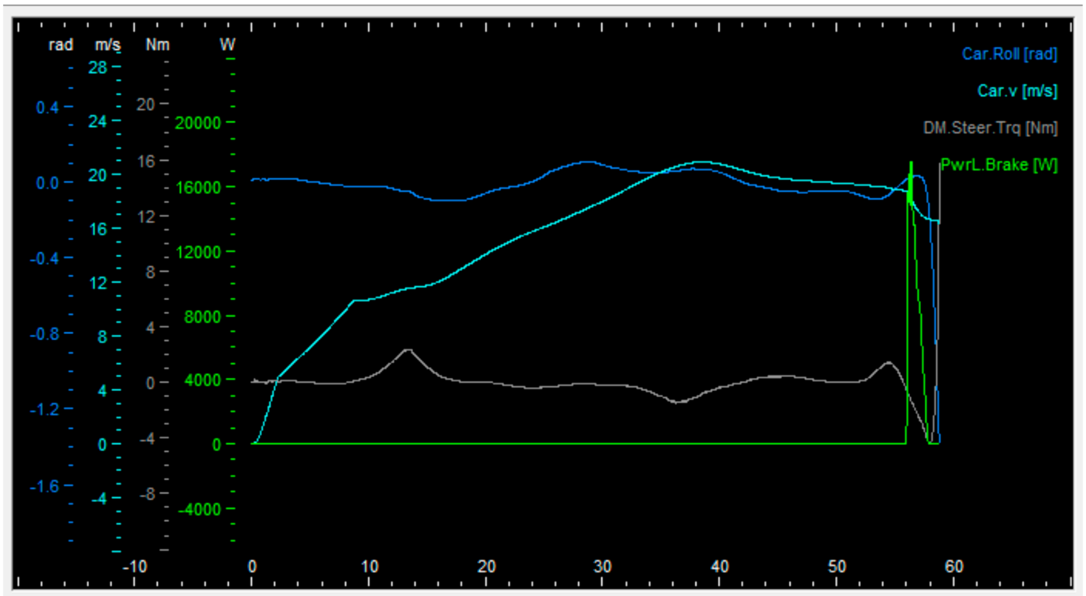

Figure 9. Carmaker® Simulation results showing the Mayfly navigating the Nordschliefe circuit.

Figure 10. Carmaker® graphic showing the Mayfly rolling during simulation.

5. Conclusions

The Mayfly's design overcomes many of the issues associated with delta three-wheeled cars, and compares very favorably to the Reliant Robin (used as a benchmark). This is largely due to the lower center of gravity, which nearly doubles the rollover threshold.

The suspension can be stiffened to give a response comparable to other sports cars, and further improve handling by reducing vehicle roll when cornering. Suggested values have been derived from a quarter car model.

The Mayfly does understeer, more so than the Robin. While this ensures stability, it does mean that the Mayfly will not have the responsive handling desirable on a sports car. Performance could be improved by moving the center of gravity rearwards, easily achieved by repositioning the battery. This will also address the tendency to 'tip' associated with delta vehicles with understeer. Other measures to improve performance include using larger wheels, stiffer tires, steering the rear wheels and torque vectoring.

Simulation, which allows the modelling of transient cornering (known to be an issue on delta vehicles) and yaw-roll interaction, shows that the Mayfly performs better than the Robin on standard tests such as ISO Lane Change and 18m slalom. However, the Mayfly still rolls over when cornering at moderate speed, which would be an issue for a sports car. Remedial action such as further widening the track and lowering the **center** of gravity, and implementing a tilting front wheel to help maintain lateral force in braked turns, can be investigated.

Note that much of the data used in these analyses is assumed, and should be updated as the design progresses.

The development of a transient model is proposed to better understand rollover and the combined pitch/roll motion unique to 'delta' three-wheeled vehicles. This will allow a more in-depth investigation of the effects of the recommended design changes.

Acknowledgments: This work was supported by an Innovation Voucher, awarded to MIBRID Ltd. by the European Regional Development Fund via the Innovation Programme for Greater Lincolnshire. The author gratefully acknowledges the support of MIBRID Ltd.

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